POLITECNICO DI MILANO

Politecnico di Milano

March 19 – 23, 2018

Course Title

Discrete and Geometric Tomography

Professor Responsible

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The course will take place at the Department of Mathematics, via Bonardi, Milano (Building *NAVE*).
On **Monday** we will meet at the ground floor at 9.00.
Lectures are at the 3° Floor

Overview

COMPUTERIZED AXIAL TOMOGRAPHY (CT)
Principles and main mathematical reconstruction model.
Examples and applications.

DISCRETE TOMOGRAPHY (DT)
Ghosts and switching components.
Ryser algorithm.
Algebraic approach.
Uniqueness models.
Uniqueness and Additivity.
Examples and Applications.

GEOMETRIC TOMOGRAPHY (GT)
Hammer's problem.
Parallel and point X-rays
Tomography for special geometric objects.

PROGRAM

Monday, Morning 9:15-13:15

Overview of the Course. A brief history of CAT. Qualitative description of the Radon transform and its inversion for X-ray image reconstruction. Applications and related problems. The origin of Geometric Tomography and of Discrete Tomography. Continuous and discrete parallel X-rays. Continuous and discrete point X-rays. Remarks and examples.
Afternoon 2:30-4:30 Discussion and exercise section.

Tuesday, Morning 9:15-13:15

Projections of lattice sets with discrete parallel X-rays. The reconstruction problem in Discrete Tomography. Ryser algorithm. Bad configurations, weakly bad configurations, switching components, ghosts. Ridge functions and additivity.

Afternoon 2:30-4:30 Discussion and exercise section.

Wednesday, Morning 9:15-13:15
Algebraic approach in a finite grid and characterization of switching components. Uniqueness models in discrete tomography. Uniqueness and additivity. Reconstruction with suitable sets of four directions. Characterization of region of interests in a finite lattice grid. Remarks on applications and examples.

Afternoon 2:30-4:30 Discussion and exercise section.

Thursday, Morning 9:15-13:15

Geometric Tomography, Hammer's problem and related uniqueness problems. Mid-point construction. U-polygons and their properties. The theorem of Gardner-McMullen in the Euclidean plane. The results of Gardner and Gritzmann in the integer lattice. Projections of convex bodies with point X-rays. The theorem of Volcic in the Euclidean plane.
P-polygons. Some results and examples in the lattice.
Afternoon 2:30-4:30 Discussion and exercise section.

Friday, Morning 9:15-12:15 *Exam Section.*

Correction, marking and discussion.

MAIN REFERENCES

1) Richard Gardner, ***Geometric Tomography***, *Cambridge University Press, New York*, second edition, 2006.
2) Gabor T. Herman and Attila Kuba Eds***., Advances in discrete tomography and its applications***, Applied and Numerical Harmonic Analysis. *Birkhäuser Boston, Inc., Boston, MA,* 2007.

Further references will be given during the course, and cited papers will be supplied to all interested students

USEFUL REFERENCES PAPERS

[1] S. van Aert, K.J. Batenburg, M.D. Rossell, R. Erni, and G. vanTendeloo, *Three-dimensional atomic imaging of crystalline nanoparticles*, Nature, 470, (2011), pp. 374-377.

[2] A. Alpers and S. Brunetti, *Stability results for the reconstruction of binary pictures from two projections*, Image and Vision Computing 25, (2007), pp. 1599–608.

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case, SIAM J. Appl. Math. 50 (1990), pp. 288-306.

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Discrete Math. 91 (1991), pp.149-159.

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Algorithms and Application, ed. by G. T. Herman and A. Kuba, Birkh¨auser, Boston, 1999, pp. 35–58.

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in: Vision Geometry V, ed. by R. A. Melter, A. Y.Wu, and L. Latecki, Society of Photo-Optical Instrumentation Engineers Proceedings

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York, 1960).

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Further references will be given during the course, and a few cited papers will be supplied.